An Overview of Complex Systems

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Physics Research Happens at the Frontiers

Very Small

Very Large

Very Complex
Contents

• Complex System
  – Neural Networks
• Reductionism
• Effective Theory and RG
  – Critical Phenomena & CFT
• Networks
• Where to from here?
What is a Complex System?

A Complex System is any system which involves

- A large number of elements, with non-linear interactions, arranged in structure(s) which can exist on many interconnected scales, with dynamics not reducible to only one level of explanation giving rise to emergent phenomena,
Complex Systems Are NOT

NOT at equilibrium,
NOT closed,
NOT integrable.
Overlap of many domains

Complex Systems

Game Theory
- Prisoner's dilemma (PD)
- Rational decision making
- Iterative PD
- n-person PD
- Bounded rationality
- Cooperation versus competition
- Spatial network game theory
- Evolutionary game theory

Collective Behavior
- Social dynamics
- Collective intelligence
- Self-organized criticality
- Agent-based modeling
- Phase transition
- Synchronization
- Ant colony optimization
- Particle swarm optimization
- Swarm behavior

Nonlinear Dynamics
- Attractors
- Phase space
- Stability analysis
- Chaos
- Population dynamics
- Multistability
- Bifurcation
- Coupled map lattices

Emergence over scale

Self-Organization over time

Systems Theory
- Homeostasis
- Feedbacks
- Self-reference
- Goal-oriented guided behavior
- Sense making
- Complexity measurement
- Information theory
- Computation theory
- Cyberscience

Networks
- Scale-free networks
- Social network analysis
- Small-world networks
- Community identification
- Centrality
- Fractals
- Scaling
- Robustness/vulnerability
- Systems biology
- Dynamical networks
- Adaptive networks

Pattern Formation
- Spatial fractals
- Reaction-diffusion systems
- Partial differential equations
- Percolation
- Cellular automata
- Spatial ecology
- Self-replication
- Spatial-evolutionary biology
- Geomorphology

Evolution & Adaptation
- Artificial neural networks
- Evolutionary computation
- Genetic algorithms/programming
- Artificial life
- Artificial intelligence
- Machine learning
- Evo-Dev
- Artificial evolution
- Evolutionary robotics
- Evolvability
Mathematical tools

• Statistical Physics, Non-equilibrium Systems
• Critical Phenomena, Conformal Field Theory
• Non-linear Dynamics, Chaos, Fractals
• Network (Graph) Theory, Random Walks
• Game Theory
• Information Theory
• ...
The earliest precursor

• Classical political economy of the Scottish Enlightenment, later developed by the Austrian school of economics, which maintains that:

“Order in market is emergent and not the execution of any human design”.


History
precursors of Complex Systems are:

1. Order in society is emergent: Adam Ferguson 1767
2. Bertalanffy 1950 Systems Theory: system refers specifically to self-regulating systems, i.e. Systems that are self-correcting through feedback.
3. Cybernetics is the interdisciplinary study of the structure of regulatory systems, closely related to information theory, control theory and systems theory, Harold S. Black 1927 electrical network theory, mechanical engineering, logic modeling,
4. Artificial Intelligence, inventing thinking machine; started in a conference on the campus of Dartmouth College 1956
Civilization is the most complex system known

Civilizations can be seen as networks of cities that emerge from pre-urban cultures, and are defined by the economic, political, military, diplomatic, social, and cultural interactions among them.
Example: Nervous System

1. Composed of many Neurons
2. Non linear interaction among the Neurons (non-linear, network)
3. Is able to learn and deduce (Emergence)
4. Many neurons fire simultaneously (Co-operation)
5. Neuron activity is continuous and active. (Non-equilibrium)
What is an Artificial Neural Network? (ANN)

- A neural network is a computational method inspired by studies of the brain and nervous systems in biological organisms.
- A Computing system made of a number of simple, highly interconnected processing elements, which process information by their dynamic state response to external input.
What can a brain do?

- Content address able memory
- Parallel Computation
- Learning
- Cognition
Neuron
Mathematical Single Neuron

Structure of a neuron in a neural net
Neural net with three neuron layers
Reductionism

• **Reductionism**: a complex system is nothing but the sum of its parts and an explanation of it can be reduced to accounts of individual constituents.

Descartes held that non-human animals could be reductively explained as automata — *De homine*, 1662.
Epiphenomena

• phenomena that can be explained completely in terms of relations between other more fundamental phenomena, are called epiphenomena.

CRP constitutes an independent cardiovascular risk factor.

C-Reactive Protein as a Cardiovascular Risk Factor More Than an Epiphenomenon.
Wim K. Lagrand et al Circulation. 1999; 100: 96-102
Superfluidity

• Is super fluidity a property of Helium Atom?

Pyotr Kapitsa and John F. Allen 1937
Many Scales

• There is a “vertical” separation of levels for example high energy and low energy- subsystems are distinct from each other.

• If interaction between two subsystems is weak one can offer effective theories at any given scale.

K. G. Wilson “Problems in Physics with Many Scales of Length” 1979, SCIENTIFIC AMERICAN
Effective Theory

• An effective theory is a scientific theory which proposes to describe a certain set of observations, but without incorporating all of the fundamental interactions

• Assumes Renormalizability, i.e. the system can be well described by moving averages, or Coarse graining.
The Renormalization Group RG

The renormalization group (RG) refers to a mathematical apparatus that allows systematic investigation of the changes of a physical system as viewed at different scales.

Block Spins

• L.P. Kadanoff (1966): "Scaling laws for Ising models near \( T_c \), Physics (Long Island City, N.Y.) 2, 263.
A Hamiltonian may be given by

$$\hat{H} = \sum_n C_n(\Lambda) \hat{O}_n$$

Where $C_n$ are coupling constants and $O_n$ are operators.

This Hamiltonian is valid only for a range of energies: $E \ll \Lambda$; hence the coupling constants are dependent on $\Lambda$:

$$\Lambda \frac{dC_n}{d\Lambda} = \beta_n(C)$$
RG Fixed Points

For certain values of the coupling constants the beta function vanishes

$$\beta_n(C^*) = 0$$

At these points the coupling constants become independent of scale: Critical Phenomena

Scale independence -> Conformal Invariance
Critical Phenomena

- Universality
- Scale invariance
- Conformal field theory

• classes of behavior in materials
• conformal invariance
Critical Phenomena

- Solid
- Liquid
- Supercritical fluid region
- Gas

Key Points:
- Melting curve
- Boiling curve
- Sublimation curve
- Critical point
- Triple point
- $P_c$
- $T_c$
Critical Exponents

- $C_v = |T-T_c|^{-\alpha}$
- $M = |T-T_c|^\beta$
- $H = M^\delta$
- $K = |T-T_c|^{-\gamma}$
- $G(p) = p^{2-\eta}$
- $\xi = |T-T_c|^{-\nu}$
\( \delta \)

Present value
\( \approx 2 \)

3D-Ising \( \approx 4.8 \)
3D-XY \( \approx 4.8 \)
3D-Heisenberg \( \approx 4.8 \)

\( \beta \)

Present value
\( \approx 1 \)

2D-Ising

3D-Ising \( \approx 0.33 \)
3D-XY \( \approx 0.35 \)
3D-Heisenberg \( \approx 0.36 \)

\( \gamma \)

Present value
\( \approx 1 \)

Mean-field

3D-Ising \( \approx 1.2 \)
3D-XY \( \approx 1.3 \)
3D-Heisenberg \( \approx 1.4 \)
Universality

Universality at the liquid-gas critical point. The liquid gas coexistence lines for a variety of atoms and small molecules, near their critical point.
Scale Invariance

• Material appears the same at different scales

• **scale invariance** is a feature of objects or laws that do not change if length scales (or energy scales) are multiplied by a common factor.
critical Ising model
Tools for Studying Critical Phenomena

- Statistical Mechanics
- Renormalization Group
- Conformal Field Theory (CFT)
  - Schramm-Loewner Evolution (SLE)
Conformal Symmetry happens in Nature when you have no Scale for example: at critical phenomena or at the Symmetry Breaking Point.

\[ m = \langle \phi \rangle \]

Free Energy is Always Convex

Continuous Transition

Abrupt Transition

Multi-valued (singular) point

Enlarge symmetry to Conformal Invariance

Spontaneous Symmetry Breaking (SSB) from G down to H
What is Conformal Invariance

Translations + Rotations + Scale + Inversion
Angles remain unchanged

\[ g_{\mu \nu} \rightarrow \Lambda(x)g_{\mu \nu} \]
30 years of conformal field theory

• Infinite conformal symmetry in two-dimensional quantum field theory, AA Belavin, AM Polyakov, AB Zamolodchikov - Nuclear Physics B, 1984

• Conformal field theory, Di Francesco, P Mathieu… - 1997

• Conformal field theory, SV Ketov - 1995

• Non-perturbative field theory: from two-dimensional conformal field theory to QCD in four dimensions, Y Frishman… - 2010

• Conformal field theory and statistical mechanics in Exact Methods in Low-Dimensional Statistical Physics, J Cardy - , 2010 – OUP-
Phase boundary in 2d Ising model honeycomb lattice is $SLE_3$
Percolation is $\text{SLE}_6$
Critical exponents from SLE

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Systems far from equilibrium may also be SLE

• Turbulence
• Surface growth (WO$_3$)
• Kardar-Parisi-Zhang Equation:
  • pinning-depinning transition
• Geometric properties of Fractals
• Self Organization
  • Sandpile model
Watershed on a smooth surface is a collection of lines along the steepest descent from maxima to saddles and back:
Consider a smooth map \( h(x,y) \), \( M \) is a 2d compact manifold

\[
h : M \rightarrow \mathbb{R}
\]

- A point \( p \) is a non-degenerate critical point of \( h \), if the derivatives of \( h \) vanish at \( p \) but its Hessian, \( H \) does not.
- The index of \( p \), \( i(p) \) is defined as the number of negative eigenvalues of \( H \) minus 1:

\[
i(p) = \# \text{negative eigenvalues of } H - 1
\]
LERW on Euclidean and Fractal landscape

- The loop-erased random walk (LERW) on the Euclidean lattice was proposed by Lawler in 1980
- Related to:
  - Abeilan sandpile model
  - q-state Potts (q→0)
  - uniform spanning trees (UST)

- Fractal dimension $d_f$ in $D = 2$ is $5/4$

- The upper critical dimension is $D = 4$, with $d_f = 2$ for $D \geq 4$
Planar $\text{LERW}_p$ & SLE

Winding angle

$V(L, p) \approx a(p) + \frac{k}{4} \ln(L)$

Direct SLE

Left passage probability

All in good agreement with SLE$_{1.73}$ !
Effective Theory

- In this sense, quantum field theory, or any other currently known physical theory, could be described as "effective", as in being the "low energy limit" of an as-yet unknown fundamental Theory.
- For example, the effective theory is applicable only for $p < \Lambda$
- This only works if you have hierarchical separable levels of explanation.
Naturalness

• Natural coefficients have the form:

\[ C_n \approx \Lambda^{d-\delta} \]

where \( \delta \) is the scaling dimension of the operator.

• Then the ratios of coupling constants depend on the cutoff energy as:

\[ \frac{C_n}{C'_n} \approx \Lambda^{-\frac{\delta}{\delta'}} \]

• An example of an unnatural theory is the Standard model the weak force is \( 10^{32} \) times larger than Gravity.
RG and Naturalness

- You need a boundary condition to solve the RG equations, usually coupling constants at low energy:

$$C_n(0) = \text{fixed}$$

- Low energy theories are easier and simpler
- Sometimes (String Theory) coupling constants are fixed at infinity.
If Naturalness does not hold, fine tuning of the physical observables will be required to explain physically observed phenomena e.g. in the Standard model the weak force is $10^{32}$ times larger than Gravity.

More technically, the question is why the Higgs boson is so much lighter than the Planck mass.
Naturalness

- If Naturalness fails an effective theory cannot exist. In other words hierarchical separation of scales is impossible.

e.g. Friction, a classical problem as yet unsolved, since the theory has to be built up right from the atoms to the meter scale.
Self Organization

How does this self-organization happen?

Standard explanation has it that the minimum of the free energy may correspond to lower entropy, but what about systems far from equilibrium?
Elements of Self Organization

1-Change occurs naturally and automatically in the systems

2-Interaction with Environmental: System not closed

3-Elements or Configurations that survive dynamics or environmental will automatically re-settle themselves, or re-organize themselves and their interactions.

4-Such responsiveness = Adaptation occurs even when the elements and system are non-organic, unintelligent, and unconscious.
The Abelian sand pile model

$1 < H_i < 5$

\[
\begin{array}{cccccc}
2 & 3 & 3 & 1 & 4 \\
2 & 3 & 1 & 3 & 4 \\
4 & 3 & 2 & 3 & 1 \\
2 & 4 & 4 & 3 & 4 \\
2 & 1 & 3 & 3 & 2 \\
\end{array}
\]
The Abelian sand pile model

\[ h_i \rightarrow h_i + 1 \]

if \( h_i > 4 \) then \( \begin{cases} h_i \rightarrow 1 \\ h_j \rightarrow h_j + 1, & j \text{ nbhd of } i \end{cases} \)
The Abelian sand pile model

$1 < H_i < 5$
The Abelian sand pile model

$1 < H_i < 5$

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The Abelian sand pile model

\[ 1 < H_i < 5 \]

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The Abelian sand pile model

Avalanches happen

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Avalanches in ASM
The Abelian sand pile model

• The total space of possible states is $4^N$
• But all states are not allowed
• Some states are recurrent and some states are transient
ASM: power law is sign of existence of many scales
C= - 2 LCFT and the Abelian sand pile model

\[ S = \frac{1}{\pi} \int \partial \theta \bar{\partial} \theta, \]

\[ \phi_{S_0}(z) = -\frac{4(\pi - 2)}{\pi^2} : \partial \theta \bar{\partial} \theta + \bar{\partial} \theta \partial \bar{\theta} :; \]

\( \theta \) is a grassmanian variable
A variety of different social, natural and technological systems can be described by the same mathematical framework.

Caldarelli, Guido
“Scale-Free Networks: Complex Webs in Nature and Technology”
Networks

Error and attack tolerance of complex networks
Réka Albert, Hawoong Jeong and Albert-László Barabási
Where to from Here?

• “Everything must be made as simple as possible. But not simpler.”
  — Albert Einstein